KCET EXAMINATION – 2023
SUBJECT : MATHEMATICS (VERSION – D3)

DATE : 20-05-2023 **TIME : 02:30 PM TO 03:50 PM**

1. If A and B are two matrices such that $AB = B$ and $BA = A$ then $A^2 + B^2 =$
(A) AB
(B) 2BA
(C) $A + B$
(D) 2AB

Ans. C

Sol. $A^2 = A.A$

$= (BA)(BA) = B(AB)A = B(BA) = BA$

$A^2 = A$

Similarly $B^2 = B$

2. If $A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$ is singular matrix, then the value of $5k - k^2$ is equal to
(A) -4
(B) 6
(C) 4
(D) -6

Ans. C

Sol. $(2-k)(3-k) - 2 = 0$

$-k^2 + 5k = 4$

3. The area of a triangle with vertices (-3,0), (3,0) and (0, k) is 9 sq.units, the value of k is
(A) 6
(B) 3
(C) 9
(D) -9

Ans. B

Sol. $\begin{vmatrix} -3 & 0 & 1 \\ 3 & 0 & 1 \\ 0 & k & 1 \end{vmatrix} = \pm 18$

$6k = \pm 18$

$k = \pm 13$

4. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ and $\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ a & b & c \end{vmatrix}$ then
(A) $\Delta_1 \neq \Delta$
(B) $\Delta_1 = -\Delta$
(C) $\Delta_1 = \Delta$
(D) $\Delta_1 = 3\Delta$

Ans. B

Sol. $\Delta_1 = \frac{1}{abc}\begin{vmatrix} a & b & c \\ abc & abc & abc \\ a^2 & b^2 & c^2 \end{vmatrix}$

$\Delta_1 = \frac{abc}{abc}\begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ a^2 & b^2 & c^2 \end{vmatrix} = \begin{vmatrix} a & 1 & a^2 \\ b & 1 & b^2 \\ c & 1 & c^2 \end{vmatrix} = -\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

$\Delta_1 = -\Delta$

5. If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ where $a, x \in (0,1)$ then the value of x is
(A) $\frac{2a}{1+a^2}$
(B) $\frac{2a}{1-a^2}$
(C) 0
(D) $\frac{a}{2}$

Ans. B

Sol. $a = \tan\alpha$, $x = \tan\theta$

$\sin^{-1}(\sin 2\alpha) + \cos^{-1}(\tan 2\alpha) = \tan^{-1}(\tan 2\theta)$

$4\alpha = 2\theta$

$2\alpha = \theta$

$2\tan^{-1} a = \tan^{-1} x$

$$\tan^{-1}\left(\frac{2a}{1-a^2}\right)=\tan^{-1}x$$

$$x=\frac{2a}{1-a^2}$$

6. The value of $\cot^{-1}\left[\frac{\sqrt{1-\sin x}+\sqrt{1+\sin x}}{\sqrt{1-\sin x}-\sqrt{1+\sin x}}\right]$ where $x\in\left(0,\frac{\pi}{4}\right)$ is

(A) $\pi-\frac{x}{3}$

(B) $\pi-\frac{x}{2}$

(C) $\frac{x}{2}$

(D) $\frac{x}{2}-\pi$

Ans. B

Sol. $\cot^{-1}\left(\frac{1-\sin x+1+\sin x+2\cos x}{1-\sin x-(1+\sin x)}\right)$

$$=\cot^{-1}\left(\frac{2+2\cos x}{-2\sin x}\right)=\cot^{-1}\left(\frac{2\cos^2\frac{x}{2}}{-2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$=\pi-\cot^{-1}\left(\cot\frac{x}{2}\right)=\pi-\frac{x}{2}$$

7. If $x\begin{bmatrix}3\\2\end{bmatrix}+y\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}15\\5\end{bmatrix}$ then the value of x and y are

(A) $x=-4,\ y=-3$

(B) $x=-4,\ y=3$

(C) $x=4,\ y=3$

(D) $x=4,\ y=-3$

Ans. C

Sol. $3x+y=15$

$2x-y=5$

$5x=20$

$x=4$ $\quad y=3$

8. If the function is $f(x)=\frac{1}{x+2}$, then the point of discontinuity of the composite function $y=f(f(x))$ is

(A) $\frac{2}{5}$

(B) $\frac{1}{2}$

(C) $-\frac{5}{2}$

(D) $\frac{5}{2}$

Ans. C

Sol. $f(f(x))=\frac{x+2}{2x+5},\quad x\neq\frac{-5}{2}$

9. If $y=a\sin x+b\cos x$, then $y^2+\left(\frac{dy}{dx}\right)^2$ is a

(A) function of x and y

(B) constant

(C) function of x

(D) function of y

Ans. B

Sol. $\frac{dy}{dx}=a\cos x-b\sin x$

$$\therefore y^2+\left(\frac{dy}{dx}\right)^2=a^2\sin^2x+b^2\cos^2x+2ab\sin x\cos x$$

$$+a^2\cos^2x+b^2\sin^2x-2ab\sin x\cos x$$

$$=a^2(\sin^2x+\cos^2x)+b^2(\cos^2x+\sin^2x)$$

$$=a^2+b^2$$

10. If

$$f(x)=1+nx+\frac{n(n-1)}{2}x^2+\frac{n(n-1)(n-2)}{6}x^3+\ldots+x^n$$

then $f''(1)=$

(A) $n(n-1)2^n$

(B) 2^{n-1}

(C) $(n-1)2^{n-1}$

(D) $n(n-1)2^{n-2}$

Ans. D

Sol. $f(x)={}^nC_0+{}^nC_1x+{}^nC_2x^2+\ldots\ldots+{}^nC_nx^n$

$f'(x)={}^nC_1+{}^nC_2(2x)+{}^nC_3(3x^2)+\ldots\ldots+{}^nC_nnx^{n-1}$

$f''(x)={}^nC_2(2)+{}^nC_3(6x)+\ldots\ldots+{}^nC_nn(n-1)x^{n-2}$

$f''(1)={}^nC_2(2)+{}^nC_3(6)+\ldots\ldots+{}^nC_nn(n-1)$

If $n=2$ then $f''(1)=2=2$

If $n=3$ then $f''(1)=3(2)+6=12$

Option verification

11. If $A=\begin{bmatrix}1 & \tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}$ and $AB=I$ then $B=$

(A) $\cos^2\frac{\alpha}{2}.I$

(B) $\sin^2\frac{\alpha}{2}.A$

(C) $\cos^2\frac{\alpha}{2}.A^T$

(D) $\cos^2\frac{\alpha}{2}.A$

Ans. C

Sol. $\begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a=\cos^2\frac{\alpha}{2}$, $b=-\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

$c=\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$, $d=\cos^2\frac{\alpha}{2}$

$\therefore B=\cos^2\frac{\alpha}{2}A^T$

12. If $u=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ and $v=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ then $\frac{du}{dv}$ is

(A) $\frac{1-x^2}{1+x^2}$

(B) 1

(C) $\frac{1}{2}$

(D) 2

Ans. B

Sol. $u=2\tan^{-1}x$ $\quad v=2\tan^{-1}x$

$\frac{du}{dx}=\frac{2}{1+x^2}$ $\quad \frac{dv}{dx}=\frac{2}{1+x^2}$

$\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=1$

13. The function $f(x)=\cot x$ is discontinuous on every point of the set

(A) $\left\{x=(2n+1)\frac{\pi}{2};\ n\in Z\right\}$

(B) $\left\{x=\frac{n\pi}{2};\ n\in Z\right\}$

(C) $\{x=n\pi;\ n\in Z\}$

(D) $\{x=2n\pi;\ n\in Z\}$

Ans. C

Sol. Concept of domain

14. A particle moves along the curve $\frac{x^2}{16}+\frac{y^2}{4}=1$. When the rate of change of abscissa is 4 times that of its ordinate, then the quadrant in which the particle lies is

(A) III or IV

(B) II or III

(C) I or III

(D) II or IV

Ans. D

Sol. $\frac{2x}{16}\frac{dx}{dt}+\frac{2y}{4}\frac{dy}{dx}=0$

$\frac{x}{8}\frac{dx}{dt}=-\frac{y}{2}\frac{dy}{dt}$

$\frac{x}{8}\frac{dy}{dt}=-\frac{y}{2}\frac{dy}{dt}$

$x=-y$

15. An enemy fighter jet is flying along the cure given by $y=x^2+2$. A solider is placed at (3,2) wants to shoot down the jet when it is nearest to him. Then the nearest distance is

(A) 2 units

(B) $\sqrt{5}$ units

(C) $\sqrt{3}$ units

(D) $\sqrt{6}$ units

Ans. B

Sol. $A(3,2)$ $\quad P(x,x^2+2)$

$AP=\sqrt{(x-3)^2+(x^2+2-2)^2}$

$=\sqrt{x^4+x^2-6x+9}$

$f'(x)=0$

$\frac{4x^3+2x-6}{\sqrt{x^4+x^2-6x+9}}=0$

$2x^3+x-3=0$

$x=1$

$AP=\sqrt{1+1+9-6}=\sqrt{5}$

16. $\int_2^8\frac{5^{\sqrt{10-x}}}{5^{\sqrt{x}}+5^{\sqrt{10-x}}}dx=$

(A) 4

(B) 3

(C) 5

(D) 6

Ans. B

Sol. $\frac{b-a}{2}=\frac{8-2}{2}=3$

17. $\int\sqrt{\operatorname{cosec}x-\sin x}\ dx=$

(A) $2\sqrt{\sin x}+C$

(B) $\frac{2}{\sqrt{\sin x}}+C$

(C) $\sqrt{\sin x}+C$

(D) $\frac{\sqrt{\sin x}}{2}+C$

Ans. A

Sol: $\int\sqrt{\frac{1-\sin^2 x}{\sin x}}dx$

$=\int\frac{\cos x}{\sqrt{\sin x}}dx$

$=2\sqrt{\sin x}+c$

18. If $f(x)$ and $g(x)$ are two functions with $g(x)=x-\frac{1}{x}$ and fog $(x)=x^3-\frac{1}{x^3}$ then $f'(x)=$

(A) $x^2-\frac{1}{x^2}$

(B) $1-\frac{1}{x^2}$

(C) $3x^2+3$

(D) $3x^2+\frac{3}{x^4}$

Ans. C

Sol: $(fog)(x)=x^3-\frac{1}{x^3}$

$=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)$

$=(g(x))^3+3g(x)$

$\Rightarrow f(x)=x^3+3x$

$f-(x)=3x^2+3$

19. A circular plate of radius 5 cm is heated. Due to expansion, its radius increases at the rate of 0.05 cm / sec. The rate at which its area is increasing when the radius is 5.2 cm is

(A) $5.05\pi cm^2$ / sec

(B) $0.52\pi cm^2$ / sec

(C) $5.2\pi cm^2$ / sec

(D) $27.4\pi cm^2$ / sec

Ans. B

Sol: $\frac{dr}{dt}=0.05$

$A=\pi r^2$

$\frac{dA}{dt}=2\pi r\frac{dr}{dt}$

$=2\pi(5.2)(0.05)$

$=0.52\pi\ cm^2/sec$

20. The distance 's' in meters travelled by a particle in 't' seconds is given by $s=\frac{2t^3}{3}-18t+\frac{5}{3}$. The acceleration when the particle comes to rest is

(A) $12\ m^2/sec$

(B) $18\ m^2/sec$

(C) $3\ m^2/sec$.

(D) $10\ m^2/sec$.

Ans. A

Sol: $v=\frac{ds}{dt}=2t^2-18=0\ \Rightarrow t=3$

$a=\frac{dv}{dt}=4t$

$=4\times3=12$

21. $\int_0^{\pi}\frac{x\tan x}{\sec x\cdot\operatorname{cosec} x}dx=$

(A) $\pi/2$

(B) $\pi^2/2$

(C) $\pi/4$

(D) $\pi^2/4$

Ans. D

Sol: $I=\int_0^{\pi}x\sin^2 x dx$

$=\int_0^{\pi}(\pi-x)\sin^2 x dx$

$2I=\pi\int_0^{\pi}\sin^2 x\,dx$

$=2\pi\frac{1}{2}\times\frac{\pi}{2}$

$=\frac{\pi^2}{2}$

$I=\frac{\pi^2}{4}$

22. $\int\sqrt{5-2x+x^2}\ dx=$

(A) $\frac{x-1}{2}\sqrt{5+2x+x^2}+2\log\left|(x-1)+\sqrt{5+2x+x^2}\right|+C$

(B) $\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|(x-1)+\sqrt{5-2x+x^2}\right|+C$

(C) $\frac{x-1}{2}\sqrt{5-2x+x^2}+2\log\left|(x+1)+\sqrt{x^2+2x+5}\right|+C$

(D) $\frac{x}{2}\sqrt{5-2x+x^2}+4\log\left|(x+1)+\sqrt{x^2-2x+5}\right|+C$

Ans. B

Sol: $\int\sqrt{4+(x-1)^2}dx$

$=\frac{x-1}{2}\sqrt{5-2x+x^2}$

$+2\log\left((x-1)+\sqrt{5-2x+x^2}\right)$

23. $\int\frac{1}{1+3\sin^2x+8\cos^2x}dx=$

(A) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

(B) $6\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

(C) $\frac{1}{6}\tan^{-1}(2\tan x)+C$

(D) $\tan^{-1}\left(\frac{2\tan x}{3}\right)+C$

Ans. A

Sol: $=\int\frac{1}{4\sin^2x+9\cos^2x}dx$

$=\int\frac{\sec^2 xdx}{4\operatorname{Tan}^2x+9}$

$=\frac{1}{4}\int\frac{\sec^2 xdx}{\operatorname{Tan}^2x+\left(\frac{3}{2}\right)^2}$

$=\frac{1}{6}\operatorname{Tan}^{-1}\left(\frac{2}{3}\operatorname{Tan}x\right)+c$

24. $\int_{-2}^{0}\left(x^3+3x^2+3x+3+(x+1)\cos(x+1)\right)dx=$

(A) 4
(B) 1
(C) 0
(D) 3

Ans. A

Sol: $\int_{-2}^{0}(x+1)^3+2+(x+1)\cos(x+1)dx$

$\frac{(x+1)^4}{4}+2x+(x+1)\sin(x+1)-\int\sin(x+1)dx$

$\left(\frac{(x+1)^4}{4}+2x+(x+1)\sin x(x+1)+\cos(x+1)\right)_{-2}^{0}$

$\frac{1}{4}+0+1(\sin 1)+\cos-\{\frac{1}{4}-4-1\sin(-1)+\cos 1\}$

$=4$

25. The degree of the differential equation

$1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2=\sqrt[3]{\frac{d^2y}{dx^2}+1}$ is

(A) 1
(B) 2
(C) 6
(D) 3

Ans. C

Sol. $\left[1+\left(\frac{dy}{dx}\right)^2+\left(\frac{d^2y}{dx^2}\right)^2\right]^3=\left[\frac{d^2y}{dx^2}+1\right]$

26. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ then

(A) $\vec{a}$ and $\vec{b}$ are coincident.

(B) Inclined to each other at 60°.

(C) $\vec{a}$ and $\vec{b}$ are perpendicular.

(D) $\vec{a}$ and $\vec{b}$ are parallel.

Ans. C

Sol. $|\bar{a}|^2+|\bar{b}|^2+2\bar{a}.\bar{b}=|\bar{a}|^2+|\bar{b}|^2+2\bar{a}.\bar{b}$

$\bar{a}.\bar{b}=0\Rightarrow\bar{a}\perp\bar{b}$

27. The component of $\hat{i}$ in the direction of the vector $\hat{i}+\hat{j}+2\hat{k}$ is

(A) $6\sqrt{6}$

(B) $\frac{\sqrt{6}}{6}$

(C) $\sqrt{6}$

(D) 6

Ans. B

Sol. $\frac{\bar{i}.(i+j+2k)}{\sqrt{1+1+4}}=\frac{1}{\sqrt{6}}$

28. In the interval $(0,\pi/2)$, area lying between the curves $y=\tan x$ and $y=\cot x$ and the X-axis is

(A) 4log2 sq. units

(B) 3log2 sq. units

(C) log2 sq. units

(D) 2log2 sq. units

Ans. C

Sol. $A=\int_0^{\frac{\pi}{4}}\tan xdx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cot xdx$

$=\log\sqrt{2}+\log\sqrt{2}=2\log\sqrt{2}$

$=\log 2$

29. The area of the region bounded by the lines $y = x + 1$ x=3 and $x = 5$ is

(A) $\frac{11}{2}$ sq. units

(B) 7. sq. units

(C) 10 sq. units

(D) $\frac{7}{2}$ sq. units

Ans. C

Sol. $A = \int_3^5 (x+1)\,dx$

$\left(\frac{x^2}{2} + x\right)_3^5 = \left(\frac{25}{2} + 5\right) - \left(\frac{9}{2} + 3\right)$

= 10

30. If a curve passes through the point $(1,1)$ and at any point (x,y) on the curve, the product of the slope of its tangent and x co-ordinate of the point is equal to the y co-ordinate of the point, then the curve also passes through the point

(A) $(-1,2)$

(B) $(\sqrt{3},0)$

(C) $(2,2)$

(D) $(3,0)$

Ans. C

Sol. $\frac{dy}{dx} \times x = y$

$\int \frac{dy}{y} = \int \frac{dx}{x} \Rightarrow y = cx$

at (1,1), c=1

$\therefore y = x$

31. The length of perpendicular drawn from the point $(3,-1,11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is

(A) $\sqrt{33}$

(B) $\sqrt{53}$

(C) $\sqrt{66}$

(D) $\sqrt{29}$

Ans. B

Sol. Let A=$(3,-1,11)$ and $P = (2t, 2+3t, 3+4t)$ be any point on the line then D.R's of AP are $(2t-3, 3+3t, 4t-8)$

Since AP perpendicular to L

$4t - 6 + 9 + 9t + 16t - 32 = 0$

$29t = 29$, t=1 $\therefore P = (2,5,7)$ then AP = $\sqrt{53}$

32. The equation of the plane through the points $(2,1,0),(3,2,-2)$ and $(3,1,7)$ is

(A) $6x - 3y + 2z - 7 = 0$

(B) $7x - 9y - z - 5 = 0$

(C) $3x - 2y + 6z - 27 = 0$

(D) $2x - 3y + 4z - 27 = 0$

Ans. B

Sol. $\begin{vmatrix} x-2 & y-1 & z \\ 1 & 1 & -2 \\ 0 & -1 & 9 \end{vmatrix} = 0 \Rightarrow 7x - 9y - z - 5 = 0$

33. The point of intersection of the line $x + 1 = \frac{y+3}{3} = \frac{-z+2}{2}$ with the plane $3x + 4y + 5z = 10$ is

(A) $(2,6,-4)$

(B) $(2,6,4)$

(C) $(-2,6,-4)$

(D) $(2,-6,-4)$

Ans. A

Sol. Let $(\lambda-1, 3\lambda-3, -2\lambda+2)$ be the general point then $3(\lambda-1) + 4(3\lambda-3) + 5(-2\lambda+2) = 10$

$\therefore \lambda = 3$ $\therefore$ point=$(2,6,-4)$

34. If $(2,3,-1)$ is the foot of the perpendicular from $(4,2,1)$ to a plane, then the equation of the plane is

(A) $2x - y + 2z = 0$

(B) $2x + y + 2z - 5 = 0$

(C) $2x - y + 2z + 1 = 0$

(D) $2x + y + 2z - 1 = 0$

Ans. C

Sol. DR's of normal to the plane are $(2,-1,2)$ $\therefore$ then equation of plane is $2(x-2) - 1(y-3) + 2(z+1) = 0$

35. $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$ then $|\vec{b}|$ is equal to

(A) 8

(B) 4

(C) 12

(D) 3

Ans. D

Sol. $|\vec{a}|^2 |\vec{b}|^2 \sin^2\theta + |\vec{a}|^2 |\vec{b}|^2 \cos^2\theta = 144$

then $16.|\vec{b}|^2 = 144 \therefore |\vec{b}| = 3$

36. If $\vec{a}+2\vec{b}+3\vec{c}=\vec{0}$ and $(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=\lambda(\vec{b}\times\vec{c})$ then the value of λ is equal to

(A) 4
(B) 6
(C) 2
(D) 3

Ans. B

Sol. $(\vec{c}\times\vec{a})=2(\vec{b}\times\vec{c})$

$a\times b=3(\vec{b}\times\vec{c})$ then

$$(\vec{a}\times\vec{b})+(\vec{b}\times\vec{c})+(\vec{c}\times\vec{a})=3(\vec{b}\times\vec{c})+(\vec{b}\times\vec{c})+2(\vec{b}\times\vec{c})$$

$\therefore \lambda=6$

37. If a line makes an angle of $\frac{\pi}{3}$ with each X and Y axis then the acute angle made by Z-axis is

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$
(D) $\frac{\pi}{3}$

Ans. B

Sol. $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\frac{1}{4}+\frac{1}{4}+\cos^2\gamma=1$ then $\cos\gamma=\pm\frac{1}{\sqrt{2}}\ \therefore \gamma=\frac{\pi}{4}$

38. Let $A=\{x,y,z,u\}$ and $B=\{a,b\}$. A function $f:A\rightarrow B$ is selected randomly. The probability that the function is an onto function is

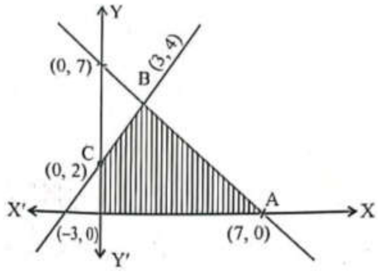
(A) $\frac{5}{8}$
(B) $\frac{1}{35}$
(C) $\frac{7}{8}$
(D) $\frac{1}{8}$

Ans. C

Sol. Number of onto functions = 2^n-2 $=16-2=14$

Total functions = $n(B)^{n(A)}=2^4=16$

39. The shaded region in the figure given is the solution of which of the inequations?



(A) $x+y\ge 7, 2x-3y+6\ge 0, x\ge 0, y\ge 0$
(B) $x+y\le 7, 2x-3y+6\le 0, x\ge 0, y\ge 0$
(C) $x+y\le 7, 2x-3y+6\ge 0, x\ge 0, y\ge 0$
(D) $x+y\ge 7, 2x-3y+6\le 0, x\ge 0, y\ge 0$

Ans. C

Sol. From diagram

40. If A and B are events such that $P(A)=\frac{1}{4}, P(A/B)=\frac{1}{2}$ and $P(B/A)=\frac{2}{3}$ then $P(B)$ is

(A) $\frac{2}{3}$
(B) $\frac{1}{2}$
(C) $\frac{1}{6}$
(D) $\frac{1}{3}$

Ans. D

Sol. $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}=\frac{2}{3}$

$\therefore P(A\cap B)=\frac{2}{3}\times\frac{1}{4}=\frac{1}{6}$

$\therefore P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)}=\frac{1}{2}$

$\therefore P(B)=2\times\frac{1}{6}=\frac{1}{3}$

41. A bag contains $2n+1$ coins. It is known that n of these coins have head on both sides whereas the other $n+1$ coins are fair. One coin is selected at random and tossed. If the probability that toss results in heads is $\frac{31}{42}$, then the value of n is

(A) 8
(B) 10
(C) 5
(D) 6

Ans. B

Sol. $\dfrac{n_{c_1}\times 1+n+1_{c1}\times\frac{1}{2}}{(2n+1)_{c1}}=\dfrac{31}{42}$

$n+\dfrac{n+1}{2}=\dfrac{31}{42}(2n+1)$

$\dfrac{3n+1}{2}=\dfrac{31(2n+1)}{42}$

$n=10$

42. The value of $\begin{vmatrix} \sin^2 14^\circ & \sin^2 66^\circ & \tan 135^\circ \\ \sin^2 66^\circ & \tan 135^\circ & \sin^2 14^\circ \\ \tan 135^\circ & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$ is

(A) 1
(B) 2
(C) -1
(D) 0

Ans. Grace

Sol. $\begin{vmatrix} \sin^2 14^\circ & \sin^2 66^\circ & -1 \\ \sin^2 66^\circ & -1 & \sin^2 14^\circ \\ -1 & \sin^2 14^\circ & \sin^2 66^\circ \end{vmatrix}$

$\sin^2 14\left(-\sin^2 66-\sin^4 14\right)-\sin^2 66\left(\sin^4 66+\sin^2 14\right)$

$-1\left(\sin^2 66.\sin^2 14-1\right)$

ORIGINAL QUESTION $\begin{vmatrix} \sin^2 24^\circ & \sin^2 66^\circ & \tan 135^\circ \\ \sin^2 66^\circ & \tan 135^\circ & \sin^2 24^\circ \\ \tan 135^\circ & \sin^2 24^\circ & \sin^2 66^\circ \end{vmatrix}$

or $\begin{vmatrix} \sin^2 14^\circ & \sin^2 76^\circ & \tan 135^\circ \\ \sin^2 76^\circ & \tan 135^\circ & \sin^2 14^\circ \\ \tan 135^\circ & \sin^2 14^\circ & \sin^2 76^\circ \end{vmatrix}$

43. The modulus of the complex number $\dfrac{(1+i)^2(1+3i)}{(2-6i)(2-2i)}$ is

(A) $\dfrac{1}{\sqrt{2}}$

(B) $\dfrac{\sqrt{2}}{4}$

(C) $\dfrac{4}{\sqrt{2}}$

(D) $\dfrac{2}{\sqrt{2}}$

Ans. **B**

Sol. $\sqrt{a+ib}=\sqrt{a^2+b^2}$

44. Given that a,b and x are real numbers and $a<b, x<0$ then

(A) $\dfrac{a}{x}<\dfrac{b}{x}$

(B) $\dfrac{a}{x}\le\dfrac{b}{x}$

(C) $\dfrac{a}{x}>\dfrac{b}{x}$

(D) $\dfrac{a}{x}\ge\dfrac{b}{x}$

Ans. **C**

Sol. $a<b$

$\dfrac{a}{x}>\dfrac{b}{x}(x<0)$

45. Ten chairs are numbered as 1 to 10. Three women and two men wish to occupy one chair each. First the women choose the chairs marked 1 to 6, then the men choose the chairs from the remaining. The number of possible ways is

(A) ${}^6C_3\times{}^4P_2$

(B) ${}^6P_3\times{}^4C_2$

(C) ${}^6C_3\times{}^4C_2$

(D) ${}^6P_3\times{}^4P_2$

Ans. C or (Grace)

Sol. $6_{c_3}\times 4_{c_2}$ or $6_{c_3}\times 7_{c_2}$

46. Which of the following is an empty set?

(A) $\{x:x^2-9=0, x\in R\}$

(B) $\{x:x^2=x+2, x\in R\}$

(C) $\{x:x^2-1=0, x\in R\}$

(D) $\{x:x^2+1=0, x\in R\}$

Ans. **D**

Sol. $x^2+1\ne 0\ \forall\ x\in R$

47. If $f(x)=ax+b$, where a and b are integers, $f(-1)=-5$ and $f(3)=3$ then a and b are respectively

(A) 0, 2
(B) 2, 3
(C) -3, -1
(D) 2, -3

Ans. **D**

Sol. $f(-1) = -5$

$-a + b = -5$

$3a + b = 3$

$a = 2, b = -3$

48. The value of

$e^{\log_{10}\tan 1^o + \log_{10}\tan 2^o + \log_{10}\tan 3^o + ... + \log_{10}\tan 89^o}$

is

(A) $\frac{1}{e}$

(B) 1

(C) 0

(D) 3

Ans. B

Sol. $\log . \tan 1^o \tan 2^o ..... \tan 89^o$

$\log . 1^0 = 0$

$e^0 = 1$

49. A line passes through $(2,2)$ and is perpendicular to the line $3x + y = 3$. Its y-intercept is

(A) 1

(B) $\frac{4}{3}$

(C) $\frac{1}{3}$

(D) $\frac{2}{3}$

Ans. B

Sol. $x - 3y = -4$

$x = 0 \Rightarrow y = 4/3$

50. The distance between the foci of a hyperbola is 16 and its eccentricity is $\sqrt{2}$. Its equation is

(A) $2x^2 - 3y^2 = 7$

(B) $y^2 - x^2 = 32$

(C) $x^2 - y^2 = 32$

(D) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

Ans. C

Sol. $2ae = 16$

$ae = 8$

$e = \sqrt{2} \Rightarrow a = 4\sqrt{2}$; $b = 4\sqrt{2}$

$x^2 - y^2 = 32$

51. If $\lim_{x \to 0} \frac{\sin(2+x) - \sin(2-x)}{x} = A\cos B$, then the values of A and B respectively are

(A) 2, 1

(B) 1, 1

(C) 2, 2

(D) 1, 2

Ans. C

Sol. $Nr = \cos(2+x) + \cos(2-x)$

$= 2\cos 2$ for $x = 0$

52. If n is even and the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is 924 x^6, then n is equal to

(A) 12

(B) 8

(C) 10

(D) 14

Ans. A

Sol. $nC_{n/2} = 924 \Rightarrow n = 12$

53. n^{th} term of the series $1 + \frac{3}{7} + \frac{5}{7^2} + \frac{1}{7^2} + ...$ is

(A) $\frac{2n-1}{7^n}$

(B) $\frac{2n+1}{7^{n-1}}$

(C) $\frac{2n-1}{7^{n-1}}$

(D) $\frac{2n+1}{7^n}$

Ans. C

Sol. $T_1 = \frac{2(1)-1}{7^o}, T_2 = \frac{2(2)-1}{7^1}$

$\therefore T_n = \frac{2n-1}{7^{n-1}}$

54. If $p\left(\frac{1}{q} + \frac{1}{r}\right), q\left(\frac{1}{r} + \frac{1}{p}\right), r\left(\frac{1}{p} + \frac{1}{q}\right)$ are in A.P., then p, q, r

(A) are in A.P.

(B) are not in G.P.

(C) are not in A.P.

(D) are in G.P.

Ans. A

Sol. For $p = 1, q = 2, r = 3$ we get A.P.

55. Let $f : R \to R$ be defined by $f(x) = 3x^2 - 5$ and $g : R \to R$ by $g(x) = \frac{x}{x^2 + 1}$ then gof is

(A) $\dfrac{3x^2}{x^4+2x^2-4}$

(B) $\dfrac{3x^2}{9x^4+30x^2-2}$

(C) $\dfrac{3x^2-5}{9x^4-30x^2+26}$

(D) $\dfrac{3x^2-5}{9x^4-6x^2+26}$

Ans. C

Sol. $gof(x)=g\left(3x^2-5\right)$

$=\dfrac{3x^2-5}{9x^4-30x^2+26}$

56. Let the relation R be defined in N by aRb if $3a+2b=27$ then R is

(A) $\{(1,2),(3,9),(5,6),(7,3),(9,0)\}$

(B) $\{(2,1),(9,3),(6,5),(3,7)\}$

(C) $\{(1,12),(3,9),(5,6),(7,3)\}$

(D) $\left\{\left(0,\dfrac{27}{2}\right),(1,12),(3,9),(5,6),(7,3)\right\}$

Ans. C

Sol. $3a+2b=27$

$R=\{(1,12),(3,9),(5,6),(7,3)\}$

57. Let $f(x)=\sin 2x+\cos 2x$ and $f(x)=x^2-1$, then $g(f(x))$ is invertible in the domain

(A) $x\in\left[\dfrac{-\pi}{2},\dfrac{\pi}{2}\right]$

(B) $x\in\left[0,\dfrac{\pi}{4}\right]$

(C) $x\in\left[\dfrac{-\pi}{4},\dfrac{\pi}{4}\right]$

(D) $x\in\left[\dfrac{-\pi}{8},\dfrac{\pi}{8}\right]$

Ans. D

Sol. $g(f(x))=g(\sin 2x+\cos 2x)$

$=(\sin 2x+\cos 2x)^2-1$

$=2\sin 2x\cos 2x$

$=\sin 4x$

$\therefore x\in\left[\dfrac{-\pi}{8},\dfrac{\pi}{8}\right]$

58. The contrapositive of the statement "If two lines do not intersect in the same plane then they are parallel." is

(A) If two are not parallel then they do not intersect in the same plane

(B) If two lines are parallel then they do not intersect in the same plane

(C) If two lines are not parallel then they intersect in the same plane

(D) If two lines are parallel then they intersect in the same plane

Ans. C

Sol. Conceptual

59. The mean of 100 observations is 50 and their standard deviation is 5. Then the sum of squares of all observations is

(A) 250000

(B) 255000

(C) 50000

(D) 252500

Ans. D

Sol. $2525=\dfrac{\sum x_i^2}{100}$

$\Rightarrow \sum x_i^2=252500$

60. $f:R\to R$ and $g:[0,\infty)\to R$ are defined by $f(x)=x^2$ an $g(x)=\sqrt{x}$. Which one of the following is not true?

(A) $(fog)(2)=2$

(B) $(gof)(-2)=2$

(C) $(gof)(4)=4$

(D) $(fog)(-4)=4$

Ans. D

Sol. $fog(-4)=f\left[g(-4)\right]$

$g(-4)$ is not defined